Department of Geography and Earth Sciences

WRM 625

Hydrological Modeling

Lecture 5

Channel Routing

- Small catchment- channel processes (storage) not important

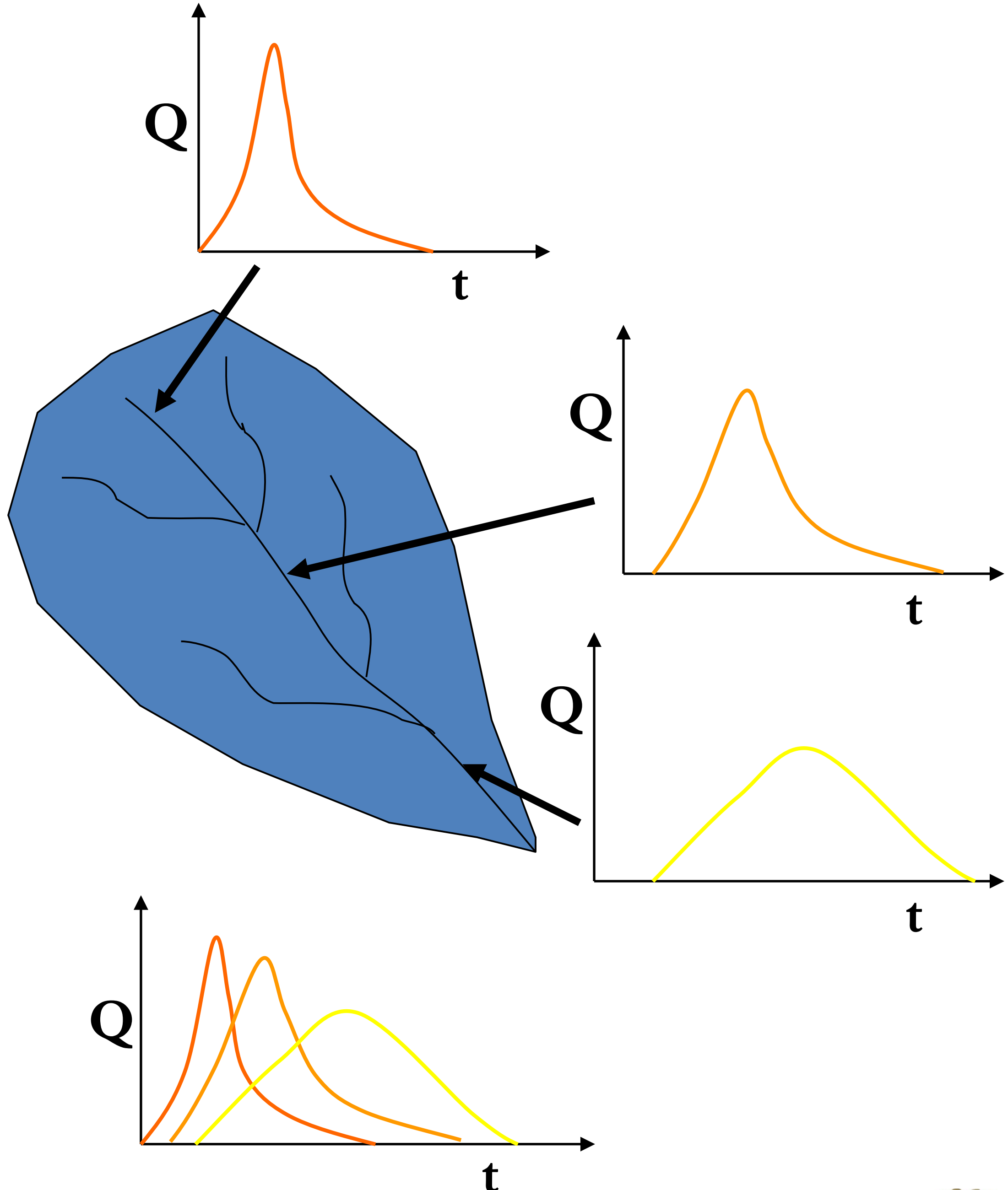
- Large catchments- Important for hydrological design as channel storage and processes become important

Channel /Flow Routing

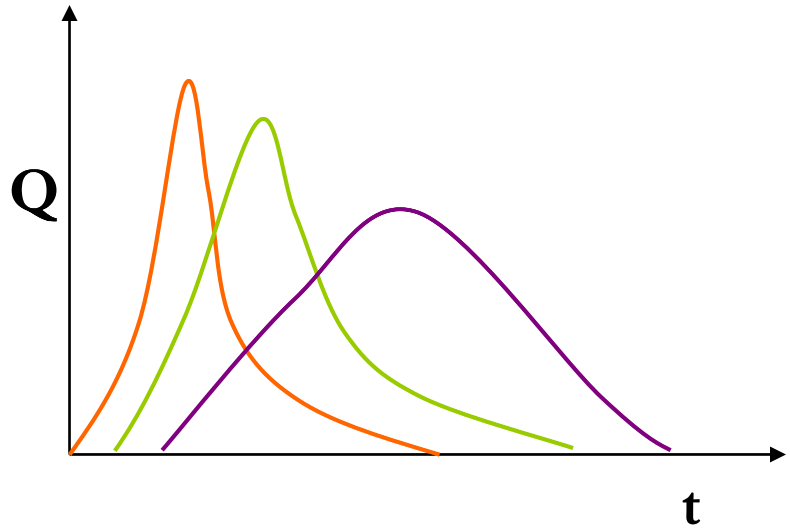
- A technique to compute the effect of system storage and system dynamics on the shape and movement of flow hydrographs along a reach or watercourse.
- When the flow is a flood, then we have flood routing.
- Routing is used to predict the temporal and spatial distribution of flood wave (or hydrograph) as it travel through various structures along a water course.

Flow Routing

- Procedure to determine the flow hydrograph at a point on a watershed from a known hydrograph upstream
- As the hydrograph travels, it
  - attenuates
  - gets delayed

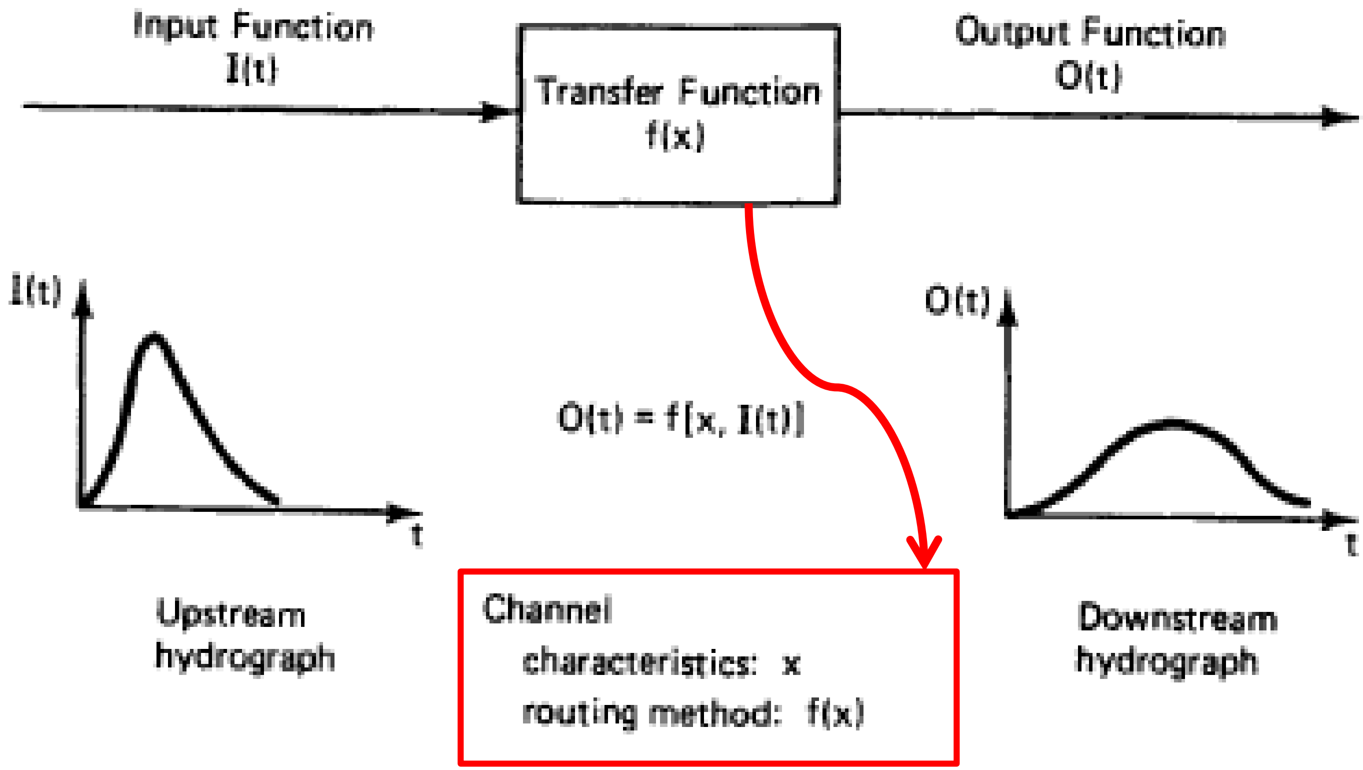


Why route flows?



- Account for changes in flow hydrograph as a flood wave passes downstream
- This helps in
  - Calculating for storages
  - Studying the attenuation of flood peaks

Systems –Theory Model

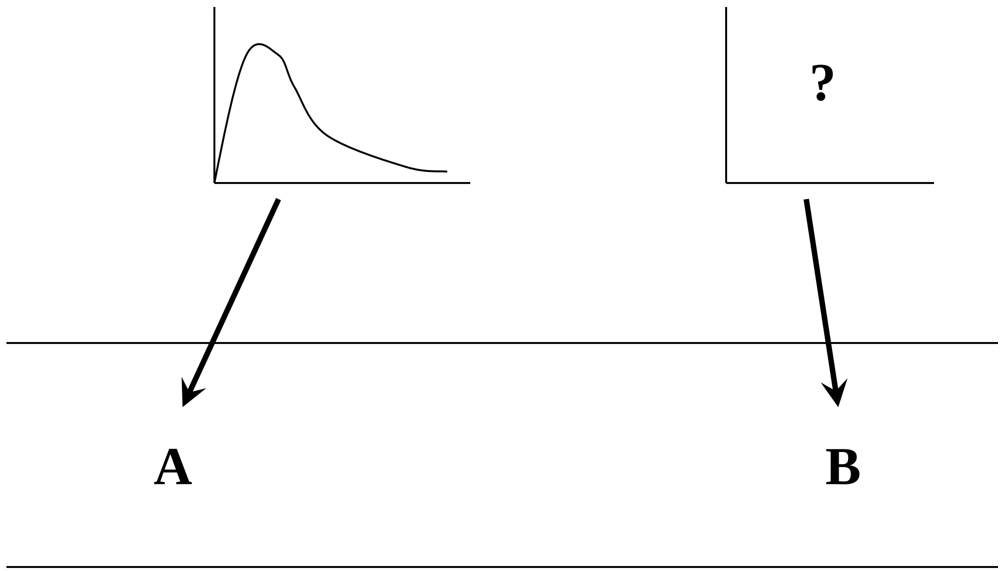


Analysis:	Known	Unknown	Known
Synthesis:	Known	Known	Unknown

Application of Flow Routing

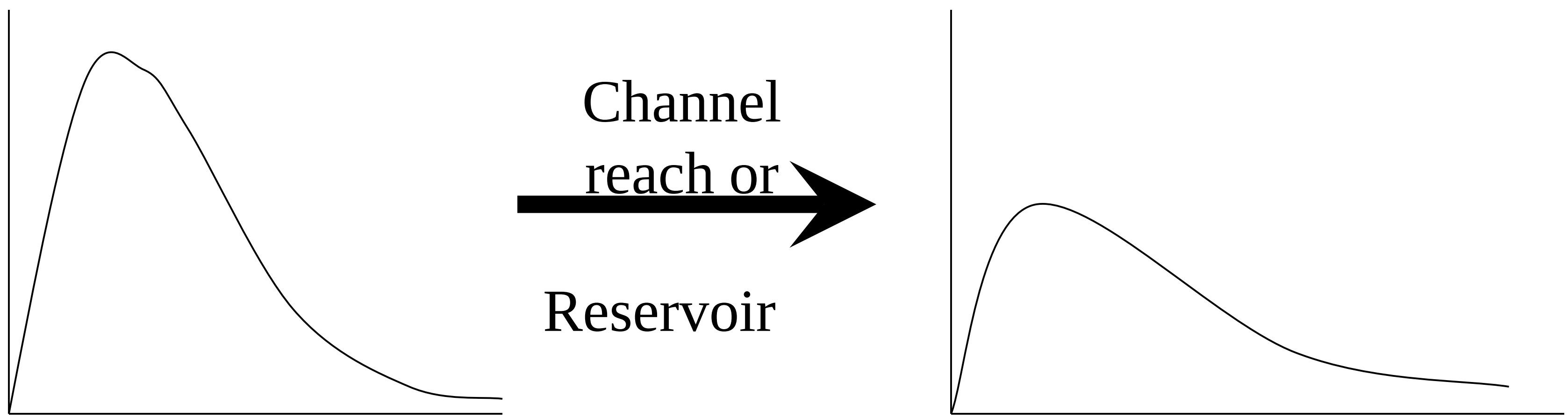
- Determine impacts of
  - Channel modifications
  - Reservoir spillway modifications
- Design structures to
  - Control storm water
  - Mitigate flood flows
  - Trap sediment
  - Flood plain delineation
- Watershed simulation

Routing Example



- Affected by
  - Slope
  - Shape
  - Roughness
  - Storage

Flow Routing



Inflow Hydrograph

Outflow Hydrograph

Classifications of Flow Routing

- **By Spatial and Temporal Variation:**

(a) **Lumped Flow Routing** – Flow is calculated as a function of time only at a fixed location in space.

(b) **Distributed Flow Routing** – Flow is calculated as a function of time and space in the system

Classifications of Flow Routing

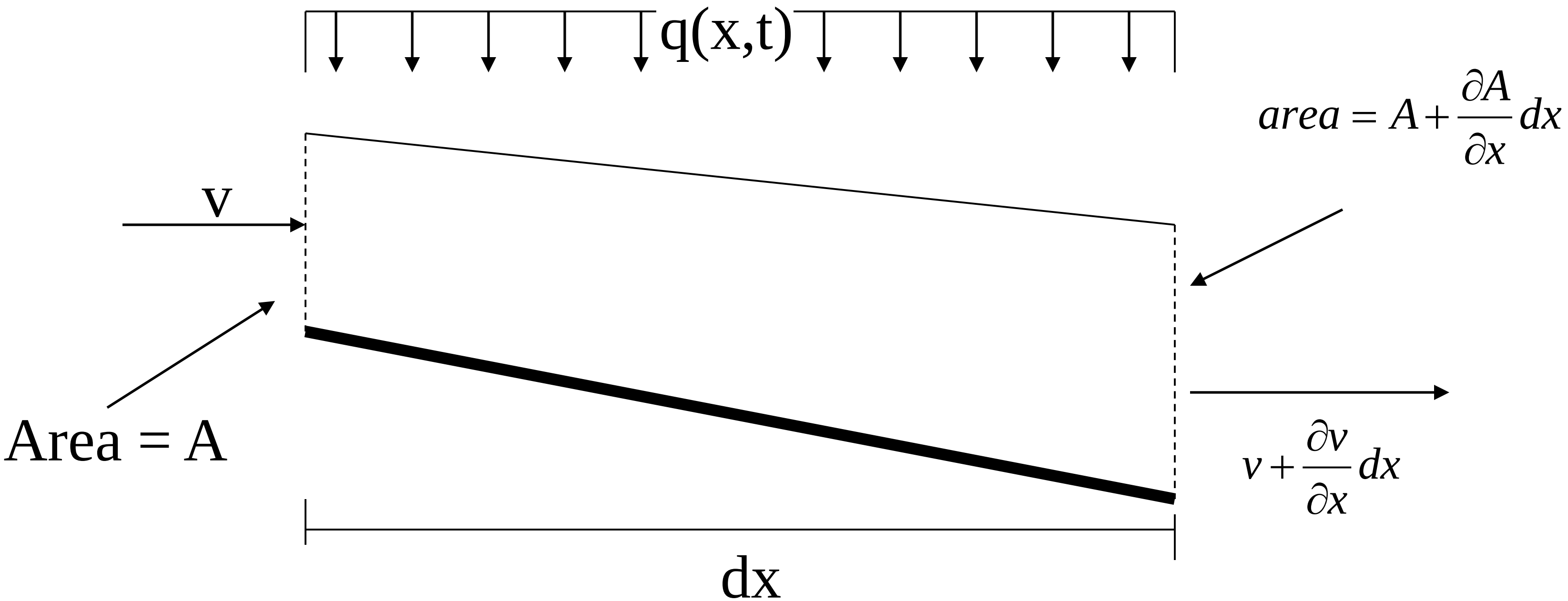
- **By Governing Equations Used:**

  (a) **Hydrologic/storage Routing** - Employs continuity equation, along with an analytical or an assumed relationship between storage and discharge within a system, in the calculation.

  - Basic storage routing
  - Muskingum routing
  - Convex Routing
  - Kinematic Routing

  (b) **Hydraulic Routing** – Use both continuity and momentum equations to describe unsteady, non-uniform flow in a flow system.

Continuity and Momentum Equations: A Review



$$\mathbf{I = O + \Delta S}$$

Classifications of Flow Routing

- **By Watercourse Type**

(a) River Flow Routing

(b) Reservoir Routing

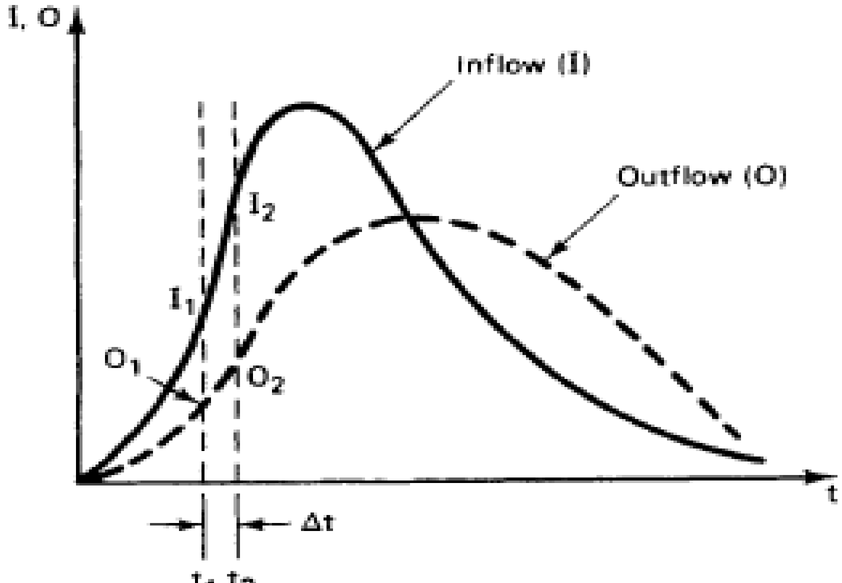
(c) Overland Flow Routing

DEVELOPMENT OF THE ROUTING EQUATION

- The Continuity Equation

$$I - O = \frac{dS}{dt} \approx \frac{\Delta S}{\Delta t}$$

- For stream flow routing, I and 0 would be the upstream and downstream discharge hydrographs.



DEVELOPMENT OF THE ROUTING EQUATION

At two different times t_1 and t2

$$\frac{I_1 + I_2}{2} - \frac{(O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t}$$

Continuity Equation (numerical form of the routing equation)

$$I_1 + I_2 + \left(\frac{2S_1}{\Delta t} - O_1 \right) = \frac{2S_2}{\Delta t} + O_2$$

Rewritten

- Assuming all inflows I and that the initial outflow and storage, O_1 and S_1, are known at t_1, then we have two unknowns O_2 and S_2

- Need a second relationship

DEVELOPMENT OF THE ROUTING EQUATION

- For steady, uniform channel flow, the inflow, outflow, and storage are a function of the depth of flow, *h.*
- From the stage-discharge relationship (that is, the rating curve) is a straight line on log-log paper, the discharges I and 0 can be represented as

$$I = ah^d \qquad S_1 = bh^m$$

$$O = ah^d \qquad S_0 = bh^m$$

•Same coefficients a and d implies that the properties of the stream reach are relatively constant;

•Same with b and m for storage characteristics

DEVELOPMENT OF THE ROUTING EQUATION

- Solving for h in I & O and substituting them into S_1 and S_o

$$S_1 = b\left[\left(\frac{I}{a}\right)^{I/d}\right]^m = b\left(\frac{I}{a}\right)^{\frac{m}{d}} = b\left[\left(\frac{1}{a}\right)^{\frac{m}{d}}\right] I^{m/d} = KI^{m/d}$$

and

$$S_0 = b\left[\left(\frac{O}{a}\right)^{I/d}\right]^m = b\left(\frac{O}{a}\right)^{\frac{m}{d}} = b\left[\left(\frac{1}{a}\right)^{\frac{m}{d}}\right] O^{m/d} = KO^{m/d}$$

Development of Routing equation

- Asuming that the weighted-average storage S is a weighted function of the storage at the upstream and downstream cross sections,

$$S = xS_1 + (1 - x)S_0$$
$$= xKI^{m/d} + (1 - x)KO^{m/d}$$

- *x is a coefficient that reflects the importance of storage at the two cross sections*

Development of Routing equation

- For uniform flow in a prismatic channel, it can be shown that d = 5/3 and m = 1; thus storage function

$$= xKI^{0.6} + (1 - x)KO^{0.6}$$

- x depends on the characteristics of the channel reach: ranges between 0.4 to 5 for natural streams and is 0 for reservoirs since storage depends only on outflow

Example

TABLE 10–1 Rating Table for USGS Gage No. 01654000

Water Year	Gage Height	Discharge	Water Year	Gage Height	Discharge
1947	9.90	3950	1968	9.46	3500
1948	5.20	780	1969	11.85	7870
1949	5.80	940	1970	7.10	1300
1950	5.74	910	1971	9.20	2380
1951	6.73	1300	1972	15.96	12000
1952	9.10	2560	1973	10.69	3270
1953	8.41	2120	1974	8.79	1890
1954	4.20	584	1975	12.90	6420
1955	9.05	2500	1976	9.14	2180
1956	7.05	1270	1977	8.91	2020
1957	6.12	990	1978	9.98	2900
1958	5.69	865	1979	11.54	4480
1959	5.99	965	1980	10.23	3100
1960	6.62	1140	1981	10.27	3170
1961	7.77	1600	1982	7.41	1190
1962	5.81	915	1983	10.22	3120
1963	8.82	2240	1984	11.31	4250
1964	7.86	1650	1985	9.03	2100
1965	7.58	1500	1986	6.49	777
1966	9.74	3400	1987	9.81	2710
1967	11.84	7870	1988	8.08	1520

The annual maximum flood series is given for the 42-yr record for a watershed located in Annandale, VA.

Example

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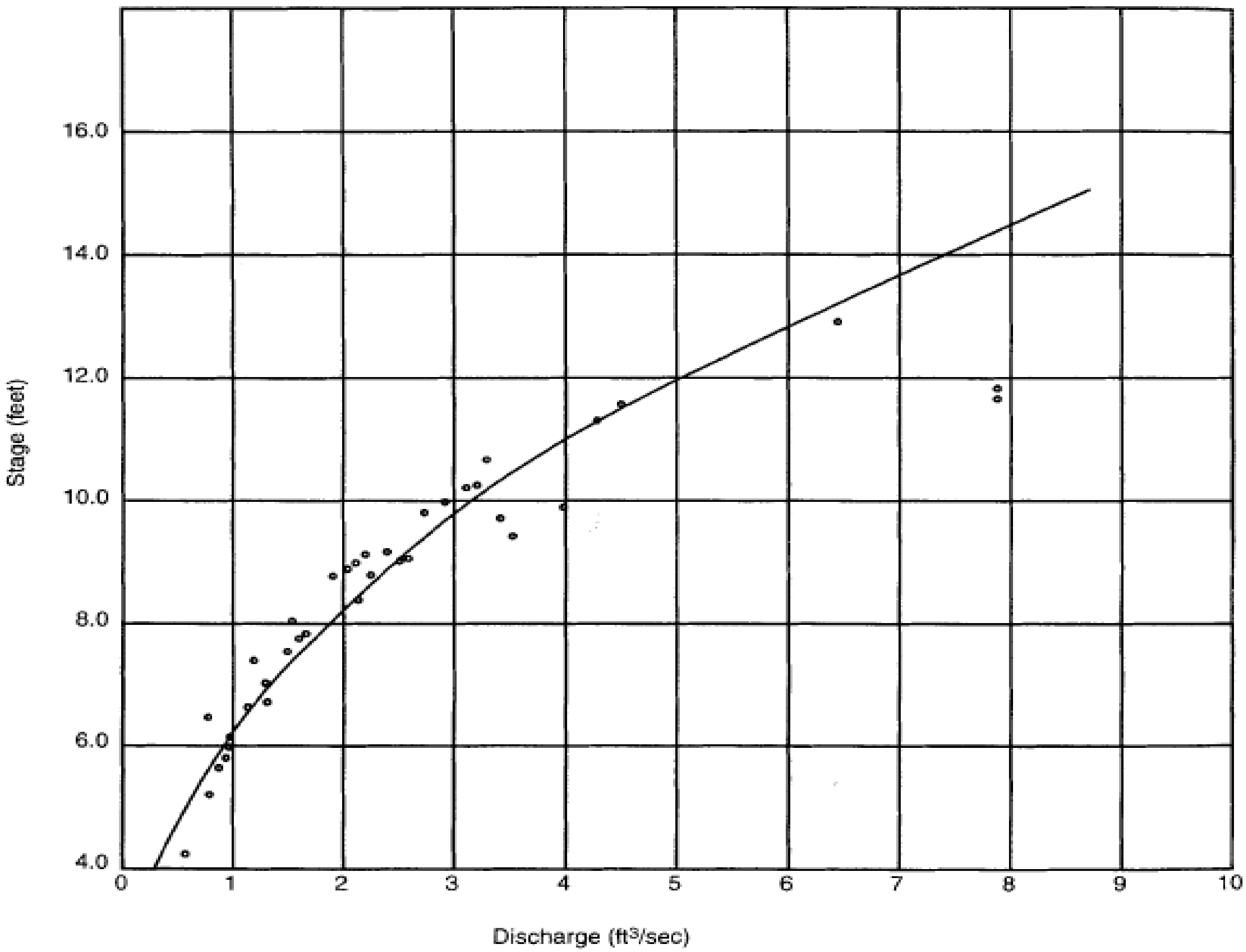
1967	11.84	7870	1988	8.08	1520

The measured stage is also given. The data were fitted using least squares analysis following a logarithmic transformation. The computed rating curve for predicting discharge for a given state is

$$Q = 11.48h^{2.44}$$

Example: rating curve



Modified Puls

- The modified puls routing method is probably most often applied to reservoir routing
- The method may also be applied to river routing for certain channel situations.
- The modified puls method is also referred to as the storage-indication method.
- The heart of the modified puls equation is found by considering the finite difference form of the continuity equation.

Modified Puls

At two different times t_1 and t2

$$\frac{I_1 + I_2}{2} - \frac{(O_1 + O_2}{2} = \frac{S_2 - S_1}{\Delta t}$$

Continuity Equation (numerical form of the routing equation-Modified Puls Model)

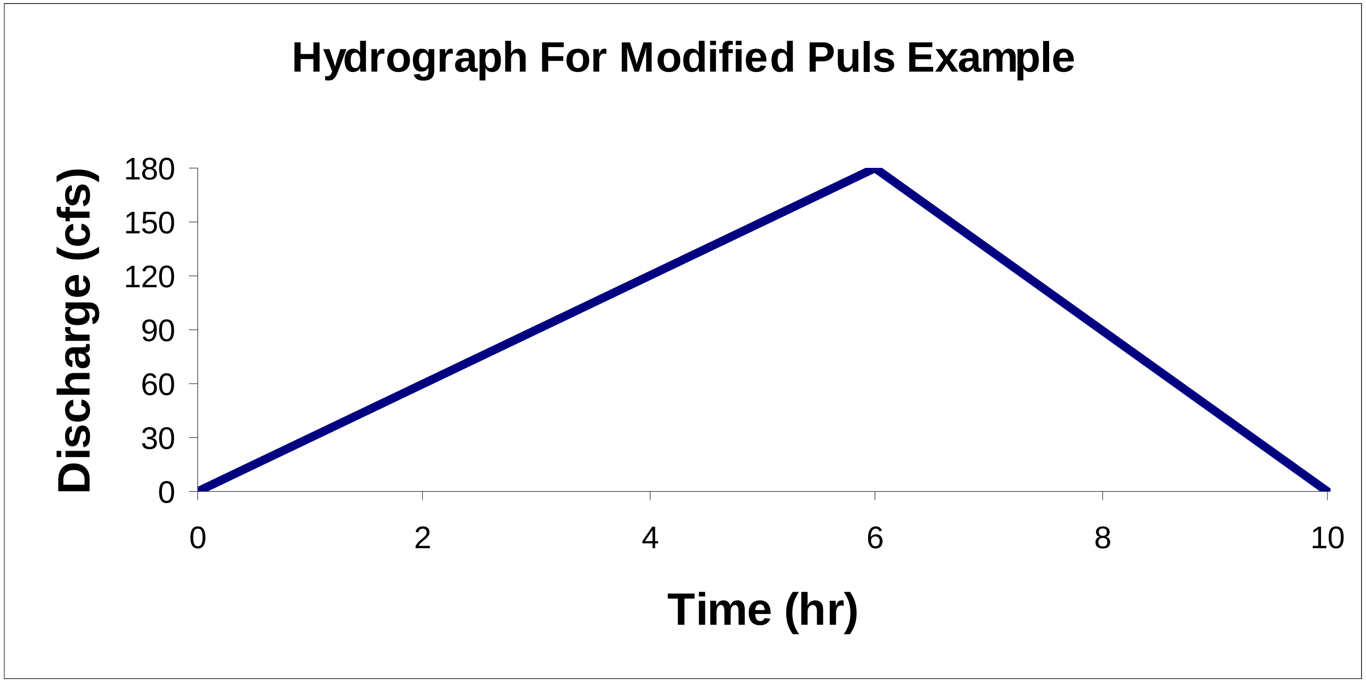
$$I_1 + I_2 + \left(\frac{2S_1}{\Delta t} - O_1 \right) = \frac{2S_2}{\Delta t} + O_2$$

Rewritten

- **The solution to the modified puls method is accomplished by developing a graph (or table) of O -vs- [2S/Δt + O].**
- **In order to do this, a stage-discharge-storage relationship must be known, assumed, or derived.**

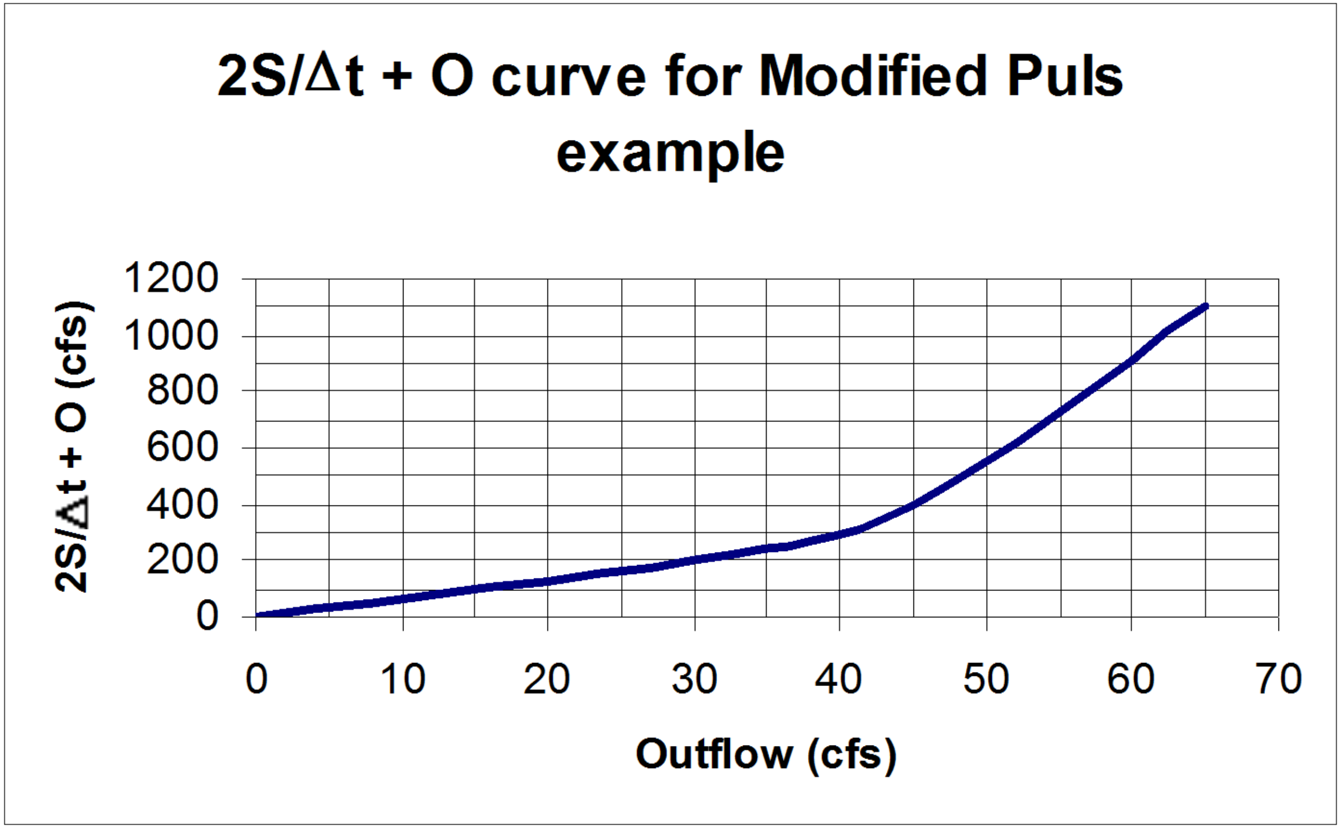
Modified Puls Example

- Given the following hydrograph and the $2S/\Delta t + O$ curve, find the outflow hydrograph for the reservoir assuming it to be completely full at the beginning of the storm.
- The following hydrograph is given:



Modified Puls Example

- The following 2S/Δt + O curve is also given:



Modified Puls Example

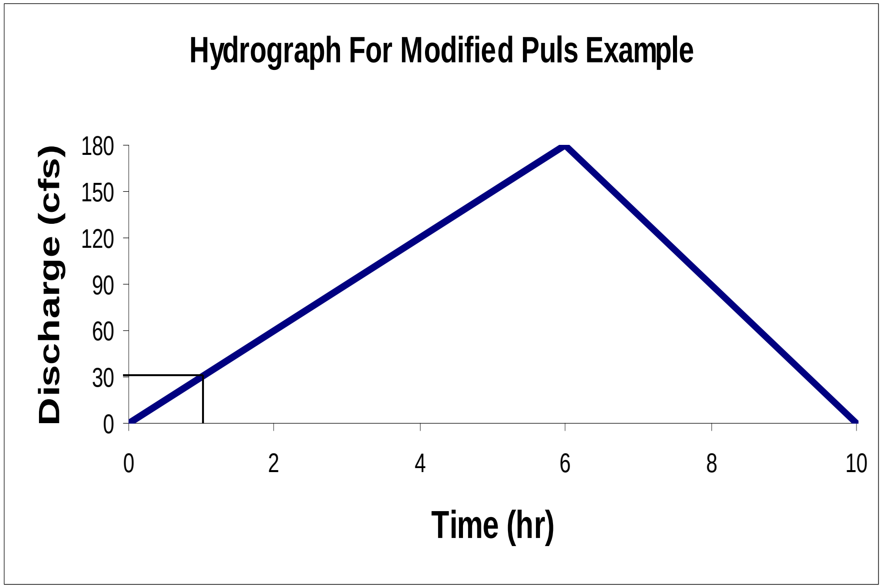
- A table may be created as follows:

Time (hr)	I_n (cfs)	I_n+I_{n+1} (cfs)	$2S_n/t - O_n$ (cfs)	$2S_n/t + O_{n+1}$ (cfs)	O_{n+1} (cfs)
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Modified Puls Example

- Next, using the hydrograph and interpolation, insert the Inflow (discharge) values.
- For example at 1 hour, the inflow is 30 cfs.

Time (hr)	I_n (cfs)	I_n+I_{n+1} (cfs)	$2S_n/t - O_n$ (cfs)	$2S_n/t + O_{n+1}$ (cfs)	O_{n+1} (cfs)
0	0				
1	30				
2	60				
3	90				
4	120				
5	150				
6	180				
7	135				
8	90				
9	45				
10	0				
11	0				
12	0				



Modified Puls Example

- The next step is to add the inflow to the inflow in the next time step.
- For the first blank the inflow at 0 is added to the inflow at 1 hour to obtain a value of 30.

Time (hr)	I_n (cfs)	I_n+I_{n+1} (cfs)	$2S_n/t - O_n$ (cfs)	$2S_n/t + O_{n+1}$ (cfs)	O_{n+1} (cfs)
0	0	30			
1	30				
2	60				
3	90				
4	120				
5	150				
6	180				
7	135				
8	90				
9	45				
10	0				
11	0				
12	0				

Modified Puls Example

- This is then repeated for the rest of the values in the column.

Time (hr)	I_n (cfs)	I_n+I_{n+1} (cfs)	$2S_n/t - O_n$ (cfs)	$2S_n/t + O_{n+1}$ (cfs)	O_{n+1} (cfs)
0	0	30			
1	30	90			
2	60	150			
3	90	210			
4	120	270			
5	150	330			
6	180	315			
7	135	225			
8	90	135			
9	45	45			
10	0	0			
11	0	0			
12	0	0			

Modified Puls Example

•The $2S_n/\Delta t + O_{n+1}$ column can then be calculated using the following equation:

$$I_1 + I_2 + \left(\frac{2S_1}{\Delta t} - O_1 \right) = \frac{2S_2}{\Delta t} + O_2$$

Note that $2S_n/\Delta t - O_n$ and O_{n+1} are set to zero.

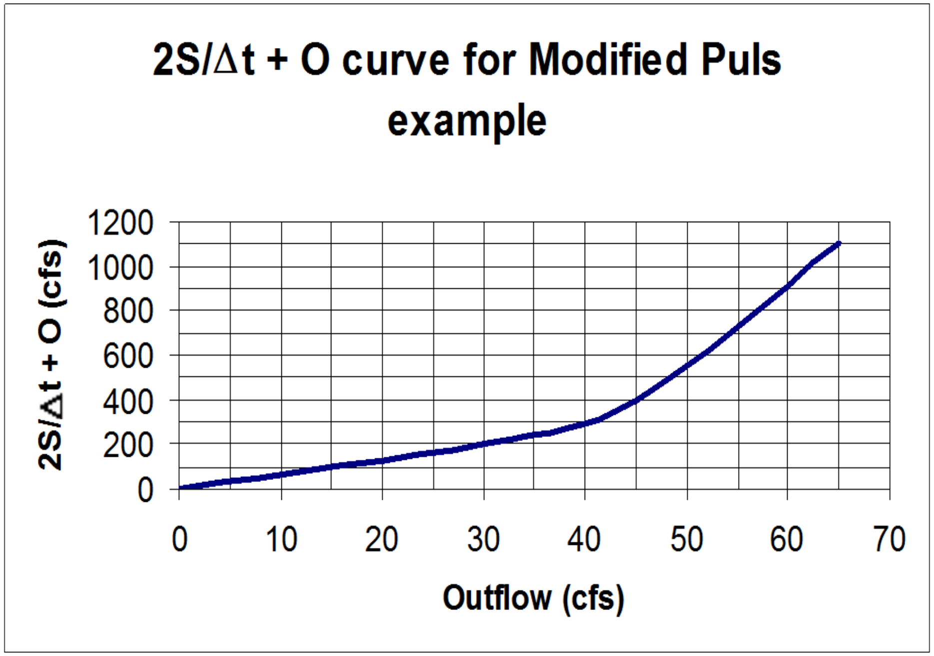
Time (hr)	I_n (cfs)	I_n+I_{n+1} (cfs)	$2S_n/t - O_n$ (cfs)	$2S_n/t + O_{n+1}$ (cfs)	O_{n+1} (cfs)
0	0	30	0		0
1	30	90		30	
2	60	150			
3	90	210			
4	120	270			
5	150	330			
6	180	315			
7	135	225			
8	90	135			
9	45	45			
10	0	0			
11	0	0			
12	0	0			

$30 + 0 = 2S_n/\Delta t + O_{n+1}$

Modified Puls Example

- Then using the curve provided outflow can be determined.
- In this case, since $2S_n/\Delta t + O_{n+1} = 30$, outflow = 5 based on the graph provided.

Time (hr)	I_n (cfs)	I_n+I_{n+1} (cfs)	$2S_n/t - O_n$ (cfs)	$2S_n/t + O_{n+1}$ (cfs)	O_{n+1} (cfs)
0	0	30	0		0
1	30	90		30	5
2	60	150			
3	90	210			
4	120	270			
5	150	330			
6	180	315			
7	135	225			
8	90	135			
9	45	45			
10	0	0			
11	0	0			
12	0	0			



Modified Puls Example

- To obtain the final column, $2S_n/\Delta t - O_n$, two times the outflow is subtracted from $2S_n/\Delta t + O_{n+1}$.
- In this example 30 - 2*5 = 20

Time (hr)	I_n (cfs)	I_n+I_{n+1} (cfs)	$2S_n/t - O_n$ (cfs)	$2S_n/t + O_{n+1}$ (cfs)	O_{n+1} (cfs)
0	0	30	0		0
1	30	90	20	30	5
2	60	150			
3	90	210			
4	120	270			
5	150	330			
6	180	315			
7	135	225			
8	90	135			
9	45	45			
10	0	0			
11	0	0			
12	0	0			

Modified Puls Example

- The same steps are repeated for the next line.
- First 90 + 20 = 110.
- From the graph, 110 equals an outflow value of 18.
- Finally 110 - 2*18 = 74

Time (hr)	I_n (cfs)	I_n+I_{n+1} (cfs)	$2S_n/t - O_n$ (cfs)	$2S_n/t + O_{n+1}$ (cfs)	O_{n+1} (cfs)
0	0	30	0		0
1	30	90	20	30	5
2	60	150	74	110	18
3	90	210			
4	120	270			
5	150	330			
6	180	315			
7	135	225			
8	90	135			
9	45	45			
10	0	0			
11	0	0			
12	0	0			

Modified Puls Example

•This process can then be repeated for the rest of the columns.
•Now a list of the outflow values have been calculated and the problem is complete.

Time (hr)	I_n (cfs)	I_n+I_{n+1} (cfs)	$2S_n/t - O_n$ (cfs)	$2S_n/t + O_{n+1}$ (cfs)	O_{n+1} (cfs)
0	0	30	0		0
1	30	90	20	30	5
2	60	150	74	110	18
3	90	210	160	224	32
4	120	270	284	370	43
5	150	330	450	554	52
6	180	315	664	780	58
7	135	225	853	979	63
8	90	135	948	1078	65
9	45	45	953	1085	65
10	0	0	870	998	64
11	0	0	746	870	62
12	0	0	630	746	58

Routing sequence

TABLE 8.2.3
Routing of flow through a detention reservoir by the level pool method (Example 8.2.1). The computational sequence is indicated by the arrows in the table.

Column: 1 Time index j	2 Time (min)	3 Inflow (cfs)	4 $I_j + I_{j+1}$ (cfs)	5 $\frac{2S_j}{\Delta t} - Q_j$ (cfs)	6 $\frac{2S_{j+1}}{\Delta t} + Q_{j+1}$ (cfs)	7 Outflow (cfs)
1	0	0		0.0		0.0
2	10	60	60	55.2	60.0	2.4
3	20	120	180	201.1	235.2	17.1
4	30	180	300	378.9	501.1	61.1
5	40	240	420	552.6	798.9	123.2
6	50	300	540	728.2	1092.6	182.2
7	60	360	660	927.5	1388.2	230.3
8	70	320	680	1089.0	1607.5	259.3
9	80	280	600	1149.0	1689.0	270.0
10	90	240	520	1134.3	1669.0	267.4
11	100	200	440	1064.4	1574.3	254.9
12	110	160	360	954.1	1424.4	235.2
13	120	120	280	820.2	1234.1	206.9
14	130	80	200	683.3	1020.2	168.5
15	140	40	120	555.1	803.3	124.1
16	150	0	40	435.4	595.1	79.8
17	160		0	338.2	435.4	48.6
18	170			272.8	338.2	32.7
19	180			227.3	272.8	22.8
20	190			194.9	227.3	16.2
21	200			169.7	194.9	12.6
22	210				169.7	9.8

End of lecture 5